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On explicit results at the intersection of the $\mathbb{Z}_2$ and $\mathbb{Z}_4$ orbifold subvarieties in K3 moduli space

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Abstract: We examine the recently found point of intersection between the $\mathbb{Z}_2$ and $\mathbb{Z}_4$ orbifold subvarieties in the K3 moduli space more closely. First we give an explicit identification of the coordinates of the respective $\mathbb{Z}_2$ and $\mathbb{Z}_4$ orbifold theories at this point. Secondly we construct the explicit identification of conformal field theories at this point and show the orthogonality of the two subvarieties.

Keywords: Superstring Vacua, Conformal Field Models in String Theory.

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1. Introduction

Compactifying superstring theory down to six dimensions we encounter as possibilities for compactification spaces not only the well understood sixteen dimensional moduli space of torus compactifications, but also the eighty dimensional moduli space of quantum K3 surfaces. Although the general mathematical properties of K3 surfaces are well known (see e.g. [1]), the precise physical properties of sigma models on these complex surfaces together with their quantum properties as the B-field and hence of the respective string vacua are only known for a nullset of theories in that moduli space, like Gepner or most of the orbifold models. A lot of pioneering work on the general structure of the K3 moduli space and the placement of the above mentioned special models within has been done in e.g. [2]–[7]. This paper aims at clarifying two points about the structure of the K3 moduli space.

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In [5, 6] it was shown that the subvarieties of $\mathbb{Z}_2$ and $\mathbb{Z}_4$ orbifold compactifications in the K3 moduli space intersect in one point. But the argument was given in a rather indirect way. First it made use of the identification of the quantum surface with its conformal field theory and performed the identification on the level of the corresponding conformal field theories. The two corresponding $\mathbb{Z}_2$ and $\mathbb{Z}_4$ orbifold conformal field theories were proven to be equivalent to the Gepner like model $(\hat{2})^4$, a $\mathbb{Z}_2$ orbifold of the Gepner model $(2)^4$. Secondly it inferred the identification of two of these three conformal field theories from the identification of two other specific theories via the orbifold procedure.

In this paper we first give an explicit identification of the two lattices and fourplanes corresponding to the two different orbifold conformal field theories at the point of intersection. This provides us with a geometric proof that the two quantum K3 surfaces are isomorphic. Secondly we elaborate the explicit identification of the three conformal field theories at that point and prove the orthogonality of the two subvarieties of $\mathbb{Z}_2$ and $\mathbb{Z}_4$ orbifold compactifications at their point of intersection. This is a new feature of the intersection which has not been clear up to now. As a byproduct we see and confirm properties of twistfields in $\mathbb{Z}_4$ orbifold models.

This explicit identification of the $\mathbb{Z}_2$ and $\mathbb{Z}_4$ orbifold conformal field theories at the point of intersection is important for further studies on the K3 moduli space. The main open problem in K3 moduli space is the study of the vast space of yet unknown sigma models besides the highly symmetric Gepner and orbifold models. The most promising idea to study these is to use conformal deformation theory (see e.g. [8]) starting from known theories. This should be easiest on the nontrivial geodesics spanned between the $\mathbb{Z}_2$ and $\mathbb{Z}_4$ orbifold subvarieties as these specific geodesics have known starting and endpoints. But for this endeavour we crucially need the exact relation between the coordinates on the two subvarieties given by the explicit identification in this article.

Furthermore, the orthogonality of the two subvarieties proven in this article allows to describe some of the nontrivial twistfield deformations of one type of orbifold by the well known torus type deformations of the other, at the point of intersection. This makes it possible to test and to understand more about these nontrivial deformations which are usually much harder to compute. One open problem to possibly analyse this way is the curious symmetry which we observed [9] in the dependence of the deformation of a vertex operator in an orbifold model on the conformal dimension about the point $h = 1/8$.

2. Lattices of orbifold theories

Let us first recall some facts about the structure of lattices signifying certain orbifold theories in K3 moduli space. We try to stick to the conventions of [5, 6, 7] where a much more detailed presentation is given.

The general structure of the K3 moduli space is given by

$$\mathrm{O}^+(\Gamma^{(4,20)})\backslash\mathrm{O}^+(4,20)/\mathrm{SO}(4)\times\mathrm{O}(20) \cong \mathrm{O}^+(\Gamma^{(4,20)})\backslash\mathrm{O}(4,20)/\mathrm{O}(4)\times\mathrm{O}(20)$$

where $\mathrm{O}^+(\Gamma^{(4,20)})$ signifies the discrete duality group. As we are only interested in local properties in this paper, we will only deal with the unique smooth, simply connected covering space $\mathrm{O}(4,20)/\mathrm{O}(4) \times \mathrm{O}(20)$. This is a Grassmanian space whose points can be described by a fourplane in an even selfdual 24-dimensional lattice $\Gamma^{(4,20)}$ of signature $(4,20)$. Such even selfdual lattices are known to be unique up to isometries for a given signature (m,n), if $m > 0$ and $n > 0$ (see e.g. [4]). [5, 6] deduce how to embed the torus moduli space into the K3 moduli space via the $\mathbb{Z}_n$, $n = 2,3,4,6$, orbifold procedure and thus give the explicit coordinates, i.e. fourplanes in corresponding lattices, for the points of these orbifold subvarieties in the K3 moduli space. In this parametrisation the reference lattices are given by the even $\mathbb{Z}$ cohomology of that surface. As we only need the coordinates of two special models at the point of intersection of the $\mathbb{Z}_2$ and $\mathbb{Z}_4$ orbifold subvarieties, we will only quote the lattices for $\mathbb{Z}_2$ and $\mathbb{Z}_4$ orbifold models in the following as well as the fourplanes and their orthogonal span for the two special models we consider.

2.1 Reference lattices for $\mathbb{Z}_2$ and $\mathbb{Z}_4$ orbifold models

Let $\mathcal{T}(\Lambda, B)$ signify the torus theory with torus lattice Λ and B-field B and let μ_i be the dual vectors to the generators λ_i of Λ. From any torus we can construct a K3 surface by using the orbifolding technique and blowing up the orbifold singularities. If we take the metric in the orbifold limit we will call this an orbifold surface. For any such orbifold surface X, let $\upsilon^\circ \in \mathcal{H}^0\,(X,\mathbb{Z})$ and $\upsilon \in \mathcal{H}^4\,(X,\mathbb{Z})$ be the generators of the respective one-dimensional cohomology groups, with $\langle \upsilon^\circ, \upsilon\rangle = 1$, $||\upsilon^\circ||^2 = ||\upsilon||^2 = 0$. $E_i \in \mathcal{H}^2\,(X,\mathbb{Z})$ signify the exceptional divisors of norm $||E_i||^2 = -2$ obtained by blowing up the orbifold singularities of the surface. Also let $P_{jk} := \mathrm{span}_{\mathbb{F}_2}\,(f_j, f_k)$ with $j,k \in \{1,\dots,4\}$ and f_j the jth standard basis vector.

Let $\mathcal{K}(\Lambda, B)$ signify the $\mathbb{Z}_2$ orbifold of $\mathcal{T}(\Lambda, B)$. The orbifold group acts via multiplication by -1 on $\mathbb{R}^4$. The lattice of even cohomology $\Gamma = \mathcal{H}^{\mathrm{even}}\,(X,\mathbb{Z})$ of the K3 surface X corresponding to $\mathcal{K}(\Lambda, B)$ is generated by ([5, 6, 7])

$$\begin{aligned} \hat{\upsilon} &= \sqrt{2}\,\upsilon\,, \\ \hat{\upsilon}^\circ &= \frac{1}{\sqrt{2}}\upsilon^\circ - \frac{1}{4}\sum_{i\in I} E_i + \sqrt{2}\,\upsilon \end{aligned} \tag{2.1}$$

and the sublattice $\hat{\Gamma}_{\mathbb{Z}_2}$ (using $\hat{E}_i := -\frac{1}{\sqrt{2}}\upsilon + E_i$)

$$\hat{\Gamma_{\mathbb{Z}_2}} := \mathrm{span}_{\mathbb{Z}}\left(\frac{1}{\sqrt{2}}\,\mu_j \wedge \mu_k + \frac{1}{2}\sum_{i\in P_{jk}} \hat{E}_{i+l}\,,\ \hat{E}_m\,;\ l,m \in I\right). \tag{2.2}$$

$I = (\mathbb{F}_2)^4$ parametrises the sixteen $\mathbb{Z}_2$ orbifold fixpoints which have been blown up with one exceptional divisor each.

Similarly let $\mathbb{Z}_4(\Lambda, B)$ signify the $\mathbb{Z}_4$ orbifold of $\mathcal{T}(\Lambda, B)$, where the orbifold group acts like $\mu_1 \mapsto \mu_2$, $\mu_2 \mapsto -\mu_1$, $\mu_3 \mapsto -\mu_4$, $\mu_4 \mapsto \mu_3$. The lattice of the even cohomology $\Gamma = \mathcal{H}^{\mathrm{even}}\,(Y,\mathbb{Z})$ of the corresponding K3 surface Y is generated by ([5, 6, 7])[1]

$$\hat{\upsilon} = 2\,\upsilon\,,$$

[1] We apologise for the abuse of notation using the same symbols $\hat{E}$ etc. as in the $\mathcal{K}(\Lambda, B)$ models in order to prevent a proliferation of indices. We hope that a distinction between the different models is clearly visible from the context.

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$$\hat{\upsilon}^\circ = \frac{1}{2}\upsilon^\circ - \frac{1}{4}\sum_{i\in I^{(2)}} E_i - \frac{1}{8}\sum_{i\in I^{(4)}} (3E_i^{(+)} + 4E_i^{(0)} + 3E_i^{(-)}) + 2\,\upsilon$$

and the sublattice $\hat{\Gamma}_{\mathbb{Z}_4}$ spanned by (using $\hat{E}_i := -\langle E_i, \hat{\upsilon}^\circ\rangle\,\hat{\upsilon} + E_i$)

$$\begin{aligned}
&\frac{1}{2}\,\mu_1\wedge\mu_2 + \frac{1}{2}\,\hat{E}_{(0,0,1,0)+\epsilon(1,1,0,0)} + \frac{1}{4}\sum_{i\in P_{34}\cap I^{(4)}} \hat{E}_{i+\epsilon(1,1,0,0)} \qquad \text{with } \epsilon\in\{0,1\}\,,\\
&\frac{1}{2}\,\mu_3\wedge\mu_4 - \frac{1}{2}\,\hat{E}_{(1,0,0,0)+\epsilon(0,0,1,1)} - \frac{1}{4}\sum_{i\in P_{12}\cap I^{(4)}} \hat{E}_{i+\epsilon(0,0,1,1)} \qquad \text{with } \epsilon\in\{0,1\}\,,\\
&\frac{1}{2}\,(\mu_1\wedge\mu_3 + \mu_4\wedge\mu_2) - \frac{1}{2}\sum_{i\in P_{13}} \hat{E}_{i+j} + \hat{E}_j \qquad \text{with } j\in I^{(4)}\,,\\
&\frac{1}{2}\,(\mu_1\wedge\mu_4 + \mu_2\wedge\mu_3) - \frac{1}{2}\sum_{i\in P_{14}} \hat{E}_{i+j} + \hat{E}_j \qquad \text{with } j\in I^{(4)}\,,\\
&\hat{E}_k \qquad \text{with } k\in I^{(2)}\cup I^{(4)}\,.
\end{aligned}$$

$I^{(4)} = \{0000, 0011, 1100, 1111\}$ parametrises the four $\mathbb{Z}_4$ orbifold fixpoints whose blow-up produces three exceptional divisors each, $I^{(2)} = \{0100, 0001, 0111, 1101, 0110, 0101\}$ the six $\mathbb{Z}_2$ orbifold fixpoints.

2.2 The $\mathbb{Z}_2$ orbifold $\mathcal{K}(\mathbb{Z}^4, 0)$

In the special model $\mathcal{K}(\mathbb{Z}^4, 0)$, the positive definite fourplane representing this point in moduli space is spanned by the following pairwise orthogonal lattice vectors of norm $||A_i|| = 4$ (within the above described lattice of signature $(4,20)$ for $\mathbb{Z}_2$ orbifold surfaces)

$$\begin{aligned}
A_{21} &= \sqrt{2}\,(e_1\wedge e_2 + e_3\wedge e_4)\\
A_{22} &= \sqrt{2}\,(e_1\wedge e_3 + e_4\wedge e_2)\\
A_{23} &= \sqrt{2}\,(e_1\wedge e_4 + e_2\wedge e_3)\\
A_{24} &= 2\hat{\upsilon}^\circ + \frac{1}{2}\sum_{i\in I^{(2)}} \hat{E}_i + 3\,\hat{\upsilon}\,.
\end{aligned}$$

The space orthogonal to this fourplane is likewise spanned by the following pairwise orthogonal lattice vectors of norm $||A_i|| = -4$

$$\begin{aligned}
A_1 &= \sqrt{2}\,(e_1\wedge e_2 - e_3\wedge e_4)\\
A_2 &= \sqrt{2}\,(e_1\wedge e_3 - e_4\wedge e_2)\\
A_3 &= \sqrt{2}\,(e_1\wedge e_4 - e_2\wedge e_3)\\
A_4 &= 2\,\hat{\upsilon}^\circ + \frac{1}{2}\sum_{i\in I^{(2)}} \hat{E}_i + \hat{\upsilon}\\
A_5 &= \frac{1}{2}(\hat{E}_{0000} - \hat{E}_{1100} - \hat{E}_{0011} + \hat{E}_{1111} - \hat{E}_{1010} - \hat{E}_{1001} - \hat{E}_{0110} - \hat{E}_{0101}) - \hat{\upsilon}\\
A_6 &= \frac{1}{2}(\hat{E}_{0000} - \hat{E}_{1100} - \hat{E}_{0011} + \hat{E}_{1111} + \hat{E}_{1010} + \hat{E}_{1001} + \hat{E}_{0110} + \hat{E}_{0101}) + \hat{\upsilon}
\end{aligned}$$

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$$
\begin{aligned}
A_7 &= \frac{1}{2}(\hat{E}_{0000} + \hat{E}_{1100} + \hat{E}_{0011} + \hat{E}_{1111} - \hat{E}_{1010} + \hat{E}_{1001} + \hat{E}_{0110} - \hat{E}_{0101}) + \hat{\upsilon} \\
A_8 &= \frac{1}{2}(\hat{E}_{0000} + \hat{E}_{1100} + \hat{E}_{0011} + \hat{E}_{1111} + \hat{E}_{1010} - \hat{E}_{1001} - \hat{E}_{0110} + \hat{E}_{0101}) + \hat{\upsilon} \\
A_9 &= \frac{1}{2}(\hat{E}_{1000} + \hat{E}_{0100} + \hat{E}_{0010} + \hat{E}_{0001} - \hat{E}_{1110} - \hat{E}_{1101} - \hat{E}_{1011} - \hat{E}_{0111}) \\
A_{10} &= \frac{1}{2}(\hat{E}_{1000} + \hat{E}_{0100} - \hat{E}_{0010} - \hat{E}_{0001} + \hat{E}_{1110} + \hat{E}_{1101} - \hat{E}_{1011} - \hat{E}_{0111}) \\
A_{11} &= \frac{1}{2}(\hat{E}_{1000} - \hat{E}_{0100} + \hat{E}_{0010} - \hat{E}_{0001} + \hat{E}_{1110} - \hat{E}_{1101} + \hat{E}_{1011} - \hat{E}_{0111}) \\
A_{12} &= \frac{1}{2}(\hat{E}_{1000} - \hat{E}_{0100} - \hat{E}_{0010} + \hat{E}_{0001} - \hat{E}_{1110} + \hat{E}_{1101} + \hat{E}_{1011} - \hat{E}_{0111}) \\
A_{13} &= \frac{1}{2}(\hat{E}_{0000} - \hat{E}_{1100} + \hat{E}_{0011} - \hat{E}_{1111} - \hat{E}_{1010} + \hat{E}_{1001} - \hat{E}_{0110} + \hat{E}_{0101}) \\
A_{14} &= \frac{1}{2}(\hat{E}_{0000} - \hat{E}_{1100} + \hat{E}_{0011} - \hat{E}_{1111} + \hat{E}_{1010} - \hat{E}_{1001} + \hat{E}_{0110} - \hat{E}_{0101}) \\
A_{15} &= \frac{1}{2}(\hat{E}_{0000} + \hat{E}_{1100} - \hat{E}_{0011} - \hat{E}_{1111} - \hat{E}_{1010} - \hat{E}_{1001} + \hat{E}_{0110} + \hat{E}_{0101}) \\
A_{16} &= \frac{1}{2}(\hat{E}_{0000} + \hat{E}_{1100} - \hat{E}_{0011} - \hat{E}_{1111} + \hat{E}_{1010} + \hat{E}_{1001} - \hat{E}_{0110} - \hat{E}_{0101}) \\
A_{17} &= \frac{1}{2}(\hat{E}_{1000} + \hat{E}_{0100} + \hat{E}_{0010} - \hat{E}_{0001} - \hat{E}_{1110} + \hat{E}_{1101} + \hat{E}_{1011} + \hat{E}_{0111}) + \hat{\upsilon} \\
A_{18} &= \frac{1}{2}(\hat{E}_{1000} + \hat{E}_{0100} - \hat{E}_{0010} + \hat{E}_{0001} + \hat{E}_{1110} - \hat{E}_{1101} + \hat{E}_{1011} + \hat{E}_{0111}) + \hat{\upsilon} \\
A_{19} &= \frac{1}{2}(\hat{E}_{1000} - \hat{E}_{0100} + \hat{E}_{0010} + \hat{E}_{0001} + \hat{E}_{1110} + \hat{E}_{1101} - \hat{E}_{1011} + \hat{E}_{0111}) + \hat{\upsilon} \\
A_{20} &= \frac{1}{2}(\hat{E}_{1000} - \hat{E}_{0100} - \hat{E}_{0010} - \hat{E}_{0001} - \hat{E}_{1110} - \hat{E}_{1101} - \hat{E}_{1011} + \hat{E}_{0111}) - \hat{\upsilon}\,.
\end{aligned}
$$

2.3 The $\mathbb{Z}_4$ orbifold $\mathbb{Z}_4(\frac{1}{\sqrt{2}}D_4, B^*)$

Now we want to look at the special theory $\mathbb{Z}_4(\frac{1}{\sqrt{2}}D_4, B^*)$, with the lattice $D_4 = \{x \in \mathbb{Z}_4 | \sum x_i \equiv 0\ (2)\}$. The B field in this theory is given as the $\Lambda^* \otimes \mathbb{R} \to \Lambda \otimes \mathbb{R}$ map ([5, 6])

$$
B^* = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}. \tag{2.3}
$$

In this model the positive definite fourplane representing this point in moduli space is spanned by the following pairwise orthogonal lattice vectors of norm $||B_i|| = 4$ (within the above described lattice of signature $(4, 20)$ for $\mathbb{Z}_4$ orbifold surfaces)

$$
\begin{aligned}
B_{21} &= 2\,e_1 \wedge e_2 + (e_1 \wedge e_3 + e_4 \wedge e_2) - (e_1 \wedge e_4 + e_2 \wedge e_3) - \hat{\upsilon} \\
B_{22} &= (e_1 \wedge e_3 + e_4 \wedge e_2) + (e_1 \wedge e_4 + e_2 \wedge e_3) \\
B_{23} &= 2\,e_3 \wedge e_4 - (e_1 \wedge e_3 + e_4 \wedge e_2) + (e_1 \wedge e_4 + e_2 \wedge e_3) \\
B_{24} &= 4\,\hat{\upsilon}^\circ + 2\,e_1 \wedge e_2 + (e_1 \wedge e_3 + e_4 \wedge e_2) - (e_1 \wedge e_4 + e_2 \wedge e_3) + \\
&\quad + \sum_{i \in I^{(2)}} \hat{E}_i + \frac{1}{2} \sum_{i \in I^{(4)}} (3\hat{E}_i^{(+)} + 4\hat{E}_i^{(0)} + 3\hat{E}_i^{(-)}) + 4\,\hat{\upsilon}\,.
\end{aligned}
$$

The space orthogonal to this fourplane is likewise spanned by the following pairwise orthogonal lattice vectors of norm $||B_i|| = -4$

$$
\begin{aligned}
B_1 &= 2\,(e_1\wedge e_2 - e_3\wedge e_4) + (e_1\wedge e_3 + e_4\wedge e_2) - (e_1\wedge e_4 + e_2\wedge e_3)\\
B_2 &= 4\,\hat{v}^{\circ} + 2\,e_1\wedge e_2 + (e_1\wedge e_3 + e_4\wedge e_2) - (e_1\wedge e_4 + e_2\wedge e_3) +\\
&\quad + \sum_{i\in I^{(2)}} \hat{E}_i + \frac{1}{2}\sum_{i\in I^{(4)}} \left(3\hat{E}_i^{(+)} + 4\hat{E}_i^{(0)} + 3\hat{E}_i^{(-)}\right) + 3\,\hat{v}\\
B_3 &= \hat{E}_{0100} - \hat{E}_{0111}\\
B_4 &= \hat{E}_{0001} - \hat{E}_{1101}\\
B_5 &= \hat{E}_{0110} - \hat{E}_{1010}\\
B_6 &= \hat{E}_{0100} + \hat{E}_{0111} + \hat{v}\\
B_7 &= \hat{E}_{0001} + \hat{E}_{1101} + \hat{v}\\
B_8 &= \hat{E}_{0110} + \hat{E}_{1010} + \hat{v}\\
B_9 &= \frac{1}{2}\Big(\hat{E}^{(+)}_{0000} + 2\hat{E}^{(0)}_{0000} + \hat{E}^{(-)}_{0000} + \hat{E}^{(+)}_{1100} + 2\hat{E}^{(0)}_{1100} + \hat{E}^{(-)}_{1100} + \hat{E}^{(+)}_{0011} - \hat{E}^{(-)}_{0011} -\\
&\quad - \hat{E}^{(+)}_{1111} + \hat{E}^{(-)}_{1111}\Big) + \hat{v}\\
B_{10} &= \frac{1}{2}\Big(\hat{E}^{(+)}_{0000} + 2\hat{E}^{(0)}_{0000} + \hat{E}^{(-)}_{0000} + \hat{E}^{(+)}_{1100} + 2\hat{E}^{(0)}_{1100} + \hat{E}^{(-)}_{1100} - \hat{E}^{(+)}_{0011} + \hat{E}^{(-)}_{0011} +\\
&\quad + \hat{E}^{(+)}_{1111} - \hat{E}^{(-)}_{1111}\Big) + \hat{v}\\
B_{11} &= \frac{1}{2}\Big(\hat{E}^{(+)}_{0000} + 2\hat{E}^{(0)}_{0000} + \hat{E}^{(-)}_{0000} - \hat{E}^{(+)}_{1100} - 2\hat{E}^{(0)}_{1100} - \hat{E}^{(-)}_{1100} - \hat{E}^{(+)}_{0011} + \hat{E}^{(-)}_{0011} -\\
&\quad - \hat{E}^{(+)}_{1111} + \hat{E}^{(-)}_{1111}\Big)\\
B_{12} &= \frac{1}{2}\Big(\hat{E}^{(+)}_{0000} + 2\hat{E}^{(0)}_{0000} + \hat{E}^{(-)}_{0000} - \hat{E}^{(+)}_{1100} - 2\hat{E}^{(0)}_{1100} - \hat{E}^{(-)}_{1100} + \hat{E}^{(+)}_{0011} - \hat{E}^{(-)}_{0011} +\\
&\quad + \hat{E}^{(+)}_{1111} - \hat{E}^{(-)}_{1111}\Big)\\
B_{13} &= \frac{1}{2}\Big(\hat{E}^{(+)}_{0000} + \hat{E}^{(-)}_{0000} + \hat{E}^{(+)}_{1100} + \hat{E}^{(-)}_{1100} + \hat{E}^{(+)}_{0011} + \hat{E}^{(-)}_{0011} + \hat{E}^{(+)}_{1111} + \hat{E}^{(-)}_{1111}\Big) + \hat{v}\\
B_{14} &= \frac{1}{2}\Big(\hat{E}^{(+)}_{0000} + \hat{E}^{(-)}_{0000} - \hat{E}^{(+)}_{1100} - \hat{E}^{(-)}_{1100} + \hat{E}^{(+)}_{0011} + \hat{E}^{(-)}_{0011} - \hat{E}^{(+)}_{1111} - \hat{E}^{(-)}_{1111}\Big)\\
B_{15} &= \frac{1}{2}\Big(\hat{E}^{(+)}_{0000} + \hat{E}^{(-)}_{0000} + \hat{E}^{(+)}_{1100} + \hat{E}^{(-)}_{1100} - \hat{E}^{(+)}_{0011} - \hat{E}^{(-)}_{0011} - \hat{E}^{(+)}_{1111} - \hat{E}^{(-)}_{1111}\Big)\\
B_{16} &= \frac{1}{2}\Big(\hat{E}^{(+)}_{0000} + \hat{E}^{(-)}_{0000} - \hat{E}^{(+)}_{1100} - \hat{E}^{(-)}_{1100} - \hat{E}^{(+)}_{0011} - \hat{E}^{(-)}_{0011} + \hat{E}^{(+)}_{1111} + \hat{E}^{(-)}_{1111}\Big)\\
B_{17} &= \frac{1}{2}\Big(\hat{E}^{(+)}_{0000} - \hat{E}^{(-)}_{0000} + \hat{E}^{(+)}_{1100} - \hat{E}^{(-)}_{1100} - \hat{E}^{(+)}_{0011} - 2\hat{E}^{(0)}_{0011} - \hat{E}^{(-)}_{0011} +\\
&\quad + \hat{E}^{(+)}_{1111} + 2\hat{E}^{(0)}_{1111} + \hat{E}^{(-)}_{1111}\Big)\\
B_{18} &= \frac{1}{2}\Big(\hat{E}^{(+)}_{0000} - \hat{E}^{(-)}_{0000} + \hat{E}^{(+)}_{1100} - \hat{E}^{(-)}_{1100} + \hat{E}^{(+)}_{0011} + 2\hat{E}^{(0)}_{0011} + \hat{E}^{(-)}_{0011} -\\
&\quad - \hat{E}^{(+)}_{1111} - 2\hat{E}^{(0)}_{1111} - \hat{E}^{(-)}_{1111}\Big)\\
B_{19} &= \frac{1}{2}\Big(\hat{E}^{(+)}_{0000} - \hat{E}^{(-)}_{0000} - \hat{E}^{(+)}_{1100} + \hat{E}^{(-)}_{1100} + \hat{E}^{(+)}_{0011} + 2\hat{E}^{(0)}_{0011} + \hat{E}^{(-)}_{0011} +
\end{aligned}
$$

$$
\begin{aligned}
&\quad + \hat{E}^{(+)}_{1111} + 2\hat{E}^{(0)}_{1111} + \hat{E}^{(-)}_{1111}\Big) + \hat{\upsilon} \\
B_{20} &= \frac{1}{2}\Big(\hat{E}^{(+)}_{0000} - \hat{E}^{(-)}_{0000} - \hat{E}^{(+)}_{1100} + \hat{E}^{(-)}_{1100} - \hat{E}^{(+)}_{0011} - 2\hat{E}^{(0)}_{0011} - \hat{E}^{(-)}_{0011} - \\
&\quad - \hat{E}^{(+)}_{1111} - 2\hat{E}^{(0)}_{1111} - \hat{E}^{(-)}_{1111}\Big) - \hat{\upsilon}\,.
\end{aligned}
$$

2.4 Identification of lattice vectors at the intersection point

In [5] it was shown that the conformal field theories associated with the K3 geometries at $\mathcal{K}(\mathbb{Z}^4, 0)$ and $\mathbb{Z}_4(\frac{1}{\sqrt{2}}D_4, B^*)$ can be identified and therefore signify the same point in K3 moduli space. A direct proof that the geometries associated with these theories are indeed the same is given by the following identification of the respective lattices of both theories which also identifies the fourplanes placed in these lattices

$$
\begin{aligned}
A_1 &\cong B_6 & A_2 &\cong -B_5 \\
A_3 &\cong -B_4 & A_4 &\cong -B_{15} \\
A_5 &\cong B_2 & A_6 &\cong -B_{13} \\
A_7 &\cong B_{14} & A_8 &\cong B_{16} \\
A_9 &\cong -B_1 & A_{10} &\cong B_3 \\
A_{11} &\cong B_8 & A_{12} &\cong B_7 \\
A_{13} &\cong -B_9 & A_{14} &\cong -B_{10} \\
A_{15} &\cong B_{11} & A_{16} &\cong B_{12} \\
A_{17} &\cong -B_{17} & A_{18} &\cong -B_{18} \\
A_{19} &\cong B_{19} & A_{20} &\cong -B_{20} \\
A_{21} &\cong -B_{21} & A_{22} &\cong B_{22} \\
A_{23} &\cong B_{23} & A_{24} &\cong B_{24}\,.
\end{aligned}
\tag{2.4}
$$

As this is a highly symmetric point in moduli space this identification is certainly only one of a great variety of possible ones.

Proof. In order to prove the above statement (2.4) we first need to express the generators of one lattice, let's say the ones of the lattice $\Gamma(\mathcal{K}(\mathbb{Z}^4, 0))$ of $\mathcal{K}(\mathbb{Z}^4, 0)$, in terms of the respective basis, the A's in this case. Using the above equivalence we can, hence, find the vectors equivalent to these generators in the $\mathbb{R}$–span of the other lattice, $\text{span}_{\mathbb{R}}(B_i;\, i = 1, \ldots, 24)$. These vectors have to be shown to be lattice vectors in $\Gamma(\mathbb{Z}_4(\frac{1}{\sqrt{2}}D_4, B^*))$ again. Now, as already explained in the beginning of section 2, both lattices are known to be even selfdual of signature $(4, 20)$ and are hence unique up to isometries. As both basis A_i and B_j consist of pairwise orthogonal vectors and the identification (2.4) preserves the norm of these basis vectors, the identification preserves the scalar product on the whole lattice. Thus, as this set of vectors is known to be generators for one of the two lattices, it has to generate the other as well. We have performed the explicit calculation for the whole set of generators of $\Gamma(\mathcal{K}(\mathbb{Z}^4, 0))$ written down in (2.1) and (2.2). A choice of typical examples of this calculation can be found in appendix A.

On the other hand the fourplanes are spanned by A_{21}, A_{22}, A_{23}, and A_{24} in $\Gamma(\mathcal{K}(\mathbb{Z}^4, 0))$ and B_{21}, B_{22}, B_{23}, and B_{24} in $\Gamma(\mathbb{Z}_4(\frac{1}{\sqrt{2}}D_4, B^*))$, and are thus identified. Hence we have

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given a simultaneous isomorphism of both lattices and both fourplanes which describe the two theories $\mathcal{K}(\mathbb{Z}^4, 0)$ and $\mathbb{Z}_4(\frac{1}{\sqrt{2}}D_4, B^*)$. This proves that both these theories signify the same point in K3 moduli space. $\square$

3. Identification of CFTs at $(\hat{2})^4$ Gepner point

Let us now turn to the conformal field theoretic identification at the $(\hat{2})^4$ Gepner point. [5, 6] have shown that two orbifold CFTs, the $\mathbb{Z}_2$ orbifold $\mathcal{K}(\mathbb{Z}^4, 0)$ and the $\mathbb{Z}_4$ orbifold $\mathbb{Z}_4(\frac{1}{\sqrt{2}}D_4, B^*)$ (now regarded as CFTs), coincide with the same Gepner like theory $(\hat{2})^4$, a certain $\mathbb{Z}_2$ orbifold of the Gepner model $(2)^4$. The last identification has been achieved via first identifying the two theories $\mathcal{K}(\frac{1}{\sqrt{2}}D_4, B^*)$ and $(\tilde{2})^4$, a $(\mathbb{Z}_2)^2$ orbifold of $(2)^4$, and then showing that the same $\mathbb{Z}_2$ orbifold action transforms both theories to the above two theories $\mathbb{Z}_4(\frac{1}{\sqrt{2}}D_4, B^*)$ and $(\hat{2})^4$. We now want to give this identification explicitely.

First recall that certain superconformal field theories at $c = 6$ can be identified using the following steps if they have a specifically enlarged symmetry algebra ([5, 6]):

- The partition function has to agree in each sector.
- The current part of the holomorphic symmetry algebra $\mathcal{A}_h$, i.e. the algebra of the fields with $(h, \bar{h}) = (1, 0)$ has to agree. For the argument to work, the $\mathfrak{su}(2)_1^2$ part of the symmetry algebra which originates in the $N = 4$ supersymmetry structure has to be enhanced by at least a $\mathfrak{u}(1)^4$. Hence $\mathfrak{u}(1)^6 \subset \mathcal{A}_h$. The same applies to the antiholomorphic symmetry algebra $\mathcal{A}_{\bar{h}}$.
- Denote the U(1) currents in $\mathfrak{su}(2)_1^2$ as $J^{(1)}$, $J^{(2)}$, the U(1) currents in $\mathfrak{u}(1)^4$ as $j^1, \ldots, j^4$. Define the "bosonic" Hilbert subspace
$$\mathcal{H}_b := \left\{ |\varphi\rangle \in \mathcal{H} \,\middle|\, J_0^{(k)}|\varphi\rangle = 0 \quad \forall\, k \in \{1, 2\} \right\}.$$
Then the charge lattice w.r.t. the subalgebra $\mathfrak{u}(1)^4$
$$\Gamma_b := \left\{ \gamma \in \mathbb{R}^{d,d} \,\middle|\, \exists|\varphi\rangle \in \mathcal{H}_b :\ j_0^k|\varphi\rangle = \gamma^k|\varphi\rangle \quad \forall\, k \in \{1, \ldots, 4\} \right\}$$
has to be isomorphic to the same selfdual lattice in both theories as well. It suffices to show this agreement for a set of generators $|\varphi\rangle$ of this selfdual lattice.

The main idea of the proof of this statement ([5, 6]) is that w.r.t. the $\mathfrak{u}(1)^4$ subalgebra the bosonic part of these theories can be viewed as a toroidal theory in $d = 4$ dimensions, which is uniquely determined by its charge lattice. The identification of the partition function and the supersymmetry algebra then determine the complete supersymmetric theories to be isomorphic.

In the special case of our three theories the partition functions have already been shown to agree and been written down in [5, 6]. We now elaborate the symmetry algebra for the three theories which is enhanced by $\mathfrak{u}(1)^4$ and the lattices of $(1/4, 1/4)$ Ramond groundstate fields explicitely. The $(1/4, 1/4)$ Ramond groundstate fields generate $\mathcal{H}_b$ as they are the lowest components of the supersymmetric groundstate fields.

3.1 The symmetry algebra of $\mathcal{K}(\mathbb{Z}^4, 0)$

As we already know the supersymmetric generators in the symmetry algebra we still need to extract the additional holomorphic vertex operators with $(h, \bar{h}) = (1, 0)$ in $\mathcal{K}(\mathbb{Z}^4, 0)$. The charge lattice of $\mathcal{K}(\mathbb{Z}^4, 0)$ is given by

$$(P_r(\mu, \lambda), P_l(\mu, \lambda)) = \frac{1}{\sqrt{2}} (\mu + \lambda, \mu - \lambda) = \left(p + \frac{1}{2} w, p - \frac{1}{2} w \right),$$

with $w = \sqrt{2}\lambda \in \sqrt{2}\Lambda = \sqrt{2}\mathbb{Z}^4$ and $p = \frac{1}{\sqrt{2}}\mu \in \frac{1}{\sqrt{2}}\Lambda^* = \frac{1}{\sqrt{2}}\mathbb{Z}^4$.

Hence we get four additional holomorphic U(1) currents which are invariant under $\mathbb{Z}_2$ action $e_i \mapsto -e_i$ $i \in \{1, \dots, 4\}$ (using the convention $V_P = V_{\lambda,\mu}$)

$$U_i = V^{\text{inv}}_{e_i, e_i} = V^t_{e_i, e_i} + V^t_{-e_i, -e_i} \qquad i = 1, \dots, 4 .$$

The cocycle factors for a $\mathfrak{u}(1)^4$ algebra of holomorphic vertex operators are naturally 1. Hence we get the OPE

$$U_i(z) U_j(w) = 2\delta_{ij} (z - w)^{-2} + O(1) .$$

The total, enhanced holomorphic symmetry algebra can be diagonalised as (with the toroidal bosonic currents $j^{(k)}_{\pm} = \frac{1}{\sqrt{2}}(j^{2k-1} \pm i\, j^{2k})$ and their superpartners $\psi^{(k)}_{\pm} = \frac{1}{\sqrt{2}}(\psi^{2k-1} \pm i\, \psi^{2k})$)

$$\begin{aligned}
J' &:= \left(\psi^{(1)}_+ \psi^{(1)}_- + \psi^{(2)}_+ \psi^{(2)}_- \right) \qquad & J'^+ &:= \sqrt{2}\psi^{(1)}_+ \psi^{(2)}_+ \qquad & J'^- &:= \sqrt{2}\psi^{(2)}_- \psi^{(1)}_- \\
A' &:= \left(\psi^{(1)}_+ \psi^{(1)}_- - \psi^{(2)}_+ \psi^{(2)}_- \right) \qquad & A'^+ &:= \sqrt{2}\psi^{(1)}_+ \psi^{(2)}_- \qquad & A'^- &:= \sqrt{2}\psi^{(2)}_+ \psi^{(1)}_- \\
P' &= \frac{1}{2}(U_1 + U_2 + U_3 + U_4) \\
Q' &= \frac{1}{2}(U_1 + U_2 - U_3 - U_4) \\
R' &= \frac{1}{2}(U_1 - U_2 + U_3 - U_4) \\
S' &= \frac{1}{2}(U_1 - U_2 - U_3 + U_4) .
\end{aligned}$$

This diagonalisation of the U(1) currents has the advantage that P' and Q' are already invariant under the $\mathbb{Z}_4$ operation

$$\begin{aligned}
e_1 &\mapsto e_2 \qquad & e_2 &\mapsto -e_1 \\
e_3 &\mapsto -e_4 \qquad & e_4 &\mapsto e_3 .
\end{aligned} \tag{3.1}$$

This $\mathbb{Z}_4$ operation acts the same way on the currents j^i and their superpartners ψ^i.

3.2 The symmetry algebra of $\mathbb{Z}_4(\frac{1}{\sqrt{2}}D_4, B^*)$

Remember the definition of D_4 and B^* in (2.3). It is now easier to study the symmetry algebra of the $\mathbb{Z}_2$ orbifold $\mathcal{K}(\frac{1}{\sqrt{2}}D_4, B^*)$ first and to get the symmetry algebra of $\mathbb{Z}_4(\frac{1}{\sqrt{2}}D_4, B^*)$ by an explicit orbifold action thereafter.

Hence, let us first look for additional holomorphic vertex operators in $\mathcal{K}(\frac{1}{\sqrt{2}}D_4, B^*)$. Due to $\Lambda = \frac{1}{\sqrt{2}}D_4$ the left charges of these are given as roots out of the root system of D_4

$$(P_l, P_r) = \frac{1}{\sqrt{2}}\left(\mu - B^*\lambda + \lambda, \mu - B^*\lambda - \lambda\right) = (\alpha_i, 0) \qquad \alpha_i \in D_4\,,$$

which leads to the following cocycle factor on the root system of D_4 ([10, 11])

$$c_{\alpha_2}(-\alpha_1) = \exp\left[\frac{i}{2}\pi\alpha_1^t B^* \alpha_2\right].$$

The roots of D_4 are given as linear combinations of the unit vectors $\alpha = e_i \pm e_j$. Hence, we can change the basis of the vertex operators in the holomorphic algebra to the more useful linear combination (following [5])

$$W_{ij}^{\pm} := \frac{1}{2}\left(V_{e_i+e_j}^{\mathrm{inv}} \pm V_{e_i-e_j}^{\mathrm{inv}}\right),$$

where

$$V_{\alpha}^{\mathrm{inv}}(z) = (V_{\alpha}^{\mathrm{torus}} + V_{-\alpha}^{\mathrm{torus}})$$

is the $\mathbb{Z}_2$ invariant linear combination of the torus vertex operators V^{torus}. We directly observe the symmetry of the indices $W_{ij}^{\pm} = W_{ji}^{\pm}$.

One can now calculate the OPEs of all the $W_{ij}^{\pm}$ and rewrite these again, in order to make the full $\mathfrak{su}(2)_1^6$ symmetry visible

$$\begin{aligned}
&J := \left(\psi_+^{(1)}\psi_-^{(1)} + \psi_+^{(2)}\psi_-^{(2)}\right) && J^+ := \sqrt{2}\psi_+^{(1)}\psi_+^{(2)} && J^- := \sqrt{2}\psi_-^{(2)}\psi_-^{(1)} \\
&A := (\psi_+^{(1)}\psi_-^{(1)} - \psi_+^{(2)}\psi_-^{(2)}) && A^+ := \sqrt{2}\psi_+^{(1)}\psi_-^{(2)} && A^- := \sqrt{2}\psi_+^{(2)}\psi_-^{(1)} \\
&P := W_{14}^- + W_{23}^- && P^{\pm} := \frac{1}{\sqrt{2}}\left((W_{12}^+ + W_{34}^+) \pm i(W_{24}^- - W_{13}^-)\right) && \\
&Q := W_{12}^+ - W_{34}^+ && Q^{\pm} := \frac{1}{\sqrt{2}}\left((W_{24}^- + W_{13}^-) \pm i(W_{14}^- - W_{23}^-)\right) && \\
&R := W_{14}^+ + W_{23}^+ && R^{\pm} := \frac{1}{\sqrt{2}}\left((W_{24}^+ - W_{13}^+) \pm i(W_{12}^- + W_{34}^-)\right) && \\
&S := W_{24}^+ + W_{13}^+ && S^{\pm} := \frac{1}{\sqrt{2}}\left((W_{12}^- - W_{34}^-) \pm i(W_{14}^+ - W_{23}^+)\right), &&
\end{aligned} \tag{3.2}$$

where the $\mathfrak{su}(2)_1$ currents $\mathcal{J}$, $\mathcal{J}^{\pm}$ are normalised to fulfil the following OPEs

$$\begin{aligned}
\mathcal{J}(z)\,\mathcal{J}^{\pm}(w) &\sim \pm 2\,(z-w)^{-1}\,\mathcal{J}^{\pm}(w) \\
\mathcal{J}^+(z)\,\mathcal{J}^-(w) &\sim \pm 2\,(z-w)^{-2} + 2\,(z-w)^{-1}\,\mathcal{J}(w) \\
\mathcal{J}(z)\,\mathcal{J}(w) &\sim \pm 2\,(z-w)^{-2}\,.
\end{aligned} \tag{3.3}$$

Now, we turn to the symmetry algebra of the $\mathbb{Z}_4(\frac{1}{\sqrt{2}}D_4, B^*)$ model. [5] have shown that it is enhanced to $\mathfrak{su}(2)^2 \otimes \mathfrak{u}(1)^4$ which is just given by the symmetry currents of (3.2) which are invariant under the $\mathbb{Z}_4$ operation (3.1). These are

$$J,\, J^{\pm}\,; \qquad P,\, P^{\pm}\,; \qquad A;\, Q;\, R;\, S\,.$$

There are no new holomorphic $(1,0)$ currents in the twisted sectors.

3.3 Non–twisted groundstate $(1/4, 1/4)$ fields

The torus theory contains eight groundstate $(1/4, 1/4)$ fields in the Ramond sector. These generate the Clifford algebra of groundstates for the Ramond sector. They can be diagonalised to have non-vanishing charges only w.r.t. to one of the currents J, A (as in (3.2)) and its respective antiholomorphic counterpart

- $E_J^{\pm}$ having charges $(\pm 1, \pm 1)$ w.r.t. J, $\bar{J}$
- $F_J^{\pm}$ having charges $(\pm 1, \mp 1)$ w.r.t. J, $\bar{J}$
- $E_A^{\pm}$ having charges $(\pm 1, \pm 1)$ w.r.t. A, $\bar{A}$
- $F_A^{\pm}$ having charges $(\pm 1, \mp 1)$ w.r.t. A, $\bar{A}$.

All of these fields survive the $\mathbb{Z}_2$ orbifolding, but only six of them,

$$E_J^{\pm}\,, \quad F_J^{\pm}\,, \quad E_A^{\pm}\,,$$

survive the above $\mathbb{Z}_4$ orbifolding with the action of equation (3.1). As the two torus symmetry fields J and A are made up of free fermions (the superpartners of the bosonic currents of the torus theory) a representation for these eight groundstate fields can be written down in terms of a suitable bosonisation.

3.4 The OPE of vertex operators with groundstate twistfields

In [12, 11, 13] it was shown that the general OPE of a vertex operator of the original torus theory with a groundstate twistfield of a $\mathbb{Z}_N$ orbifold theory looks like

$$V^{\text{torus}}_{P(\mu,\lambda)}(z)\, T^l_f(w) = (z-w)^h (\bar{z}-\bar{w})^{\bar{h}} g(P_L, P_R)\, \zeta_N^{\mu(N x_f) l}\, T^l_{f'}(w) + \cdots$$

with $\zeta_N = \exp 2\pi i/N$ and the translated fixpoint

$$x_{f'} = x_f + [(1-\theta)^{-1}\lambda] = x_f \left[\frac{1}{n(f)} \sum_{k=1}^{n(f)-1} k\theta^k \lambda \right].$$

θ is the matrix of the representation of the orbifold group generator on the torus lattice, e.g. equation (3.1) for the above $\mathbb{Z}_4$ orbifold. $f = (1-\theta)\, x_f \in \Lambda$ signifies the fixpoint the twistfield lives at; $x_f \in I$ gives the location of that fixed point, where I can be taken as a subgroup of order $n(f)$ of $H_1(T, \mathbb{R})/H_1(T, \mathbb{Z})$.

The calculation in [11] makes use of the mode expansion of the fields. There the coupling parameter $g(P_L, P_R)$, which is independent of the position of the fixpoint, is determined by a careful treatment of the zero modes to be

$$g(P_L, P_R) = e^{\pi i p^t (1-\theta)^{-1} w}\; g'_l(P_L, P_R)$$

with the vertex operator coupling constant ($d/2$ signifies the complex dimension)

$$
\begin{aligned}
g_l'(P_L, P_R) &= \prod_{\mu=1}^{d/2} \delta(l\, k_\mu)^{-(h^\mu+\bar{h}^\mu)} \\
\delta(k_\mu) &= N^2 \prod_{a=1}^{N-1} \left(2\sin\frac{\pi a}{N}\right)^{-2\cos(2\pi a k_\mu)} \\
h^\mu &= \frac{1}{2}||P_L^\mu||^2\,, \qquad \bar{h}^\mu = \frac{1}{2}||P_R^\mu||^2\,.
\end{aligned}
$$

3.5 Twisted groundstate (1/4, 1/4) fields in $\mathbb{Z}_4(\frac{1}{\sqrt{2}}D_4, B^*)$

First regard the action of the additional symmetry generators on the groundstate twistfields of this theory. We have the lattice $\Lambda = \frac{1}{\sqrt{2}}D_4 = \frac{1}{\sqrt{2}}\Lambda_d$ where Λ_d will be the lattice numbering our fixpoints, i.e. $f \in \Lambda_d$. The representation of the orbifold group generator is given by

$$
\theta_4 = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix} \qquad \theta_2 = \theta_4^2 = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{3.4}
$$

3.5.1 $\mathbb{Z}_2$ twistfields

For $\mathbb{Z}_2$ twistfields we have a coupling of

$$
g^{(N=2)}(\alpha, 0) = \frac{1}{4}\, e^{\frac{1}{2}\pi i(\alpha+B^*\alpha)^t(1-\theta_2)^{-1}\alpha} = \frac{i}{4}
$$

due to $||\alpha||^2 = 2$ and the antisymmetry of B^*. Furthermore, the translation of the fixpoint in the OPE of the holomorphic vertex operator and the twistfield is given by

$$
x_f \mapsto x_f + (1-\theta_2)^{-1}\lambda = x_f + \frac{1}{2}\frac{\alpha}{\sqrt{2}}\,. \tag{3.5}
$$

Hence we get an OPE of

$$
\begin{aligned}
V_{e_i+e_j}(z)\, T_f^{(N=2)}(w) &= (V^t_{e_i+e_j}(z) + V^t_{-e_i-e_j}(z))\, T_f^{(N=2)}(w) \\
&= g^{(N=2)}(\alpha,0)(z-w)^{-1}\left(e^{i\pi(\alpha+B^*\alpha)^t(\sqrt{2}x_f)} + e^{-i\pi(\alpha+B^*\alpha)^t(\sqrt{2}x_f)}\right) \times \\
&\quad \times T_{f'}^{(N=2)}(w) \\
&= \frac{i}{2} e^{i\pi(\alpha+B^*\alpha)^t(\sqrt{2}x_f)}\, T^{(N=2)}_{[x_f+\frac{1}{2\sqrt{2}}\alpha]}(w)\,.
\end{aligned}
$$

Now using this OPE, we can diagonalise the groundstate twistfields w.r.t. the action of the enhanced symmetry generators

$$
E_P^{\pm} = \sum_{\substack{\delta_1=\delta_2 \\ \delta_3=\delta_4}} (-1)^{\delta_3} T_{\underline{\delta}}^{(N=2)} \pm i \sum_{\substack{\delta_1\neq\delta_2 \\ \delta_3\neq\delta_4}} (-1)^{\delta_2+\delta_3} T_{\underline{\delta}}^{(N=2)}
$$

$$
\begin{aligned}
F^{\pm}_{P} &= \sum_{\substack{\delta_1\neq\delta_2\\ \delta_3=\delta_4}} T^{(N=2)}_{\underline{\delta}} \pm i \sum_{\substack{\delta_1=\delta_2\\ \delta_3\neq\delta_4}} (-1)^{\delta_1} T^{(N=2)}_{\underline{\delta}} \\
E^{\pm}_{Q} &= \sum_{\substack{\delta_1=\delta_2\\ \delta_3=\delta_4}} T^{(N=2)}_{\underline{\delta}} \pm i \sum_{\substack{\delta_1\neq\delta_2\\ \delta_3=\delta_4}} (-1)^{\delta_3} T^{(N=2)}_{\underline{\delta}} \\
F^{\pm}_{Q} &= \sum_{\substack{\delta_1=\delta_2\\ \delta_3\neq\delta_4}} (-1)^{\delta_1+\delta_3} T^{(N=2)}_{\underline{\delta}} \pm i \sum_{\substack{\delta_1\neq\delta_2\\ \delta_3\neq\delta_4}} (-1)^{\delta_2} T^{(N=2)}_{\underline{\delta}} \\
E^{\pm}_{R} &= \sum_{\substack{\delta_1=\delta_2\\ \delta_3=\delta_4}} (-1)^{\delta_1} T^{(N=2)}_{\underline{\delta}} \pm i \sum_{\substack{\delta_1\neq\delta_2\\ \delta_3\neq\delta_4}} T^{(N=2)}_{\underline{\delta}} \\
F^{\pm}_{R} &= \sum_{\substack{\delta_1\neq\delta_2\\ \delta_3=\delta_4}} (-1)^{\delta_2+\delta_3} T^{(N=2)}_{\underline{\delta}} \pm i \sum_{\substack{\delta_1=\delta_2\\ \delta_3\neq\delta_4}} (-1)^{\delta_3} T^{(N=2)}_{\underline{\delta}} \\
E^{\pm}_{S} &= \sum_{\substack{\delta_1=\delta_2\\ \delta_3=\delta_4}} (-1)^{\delta_1+\delta_3} T^{(N=2)}_{\underline{\delta}} \pm i \sum_{\substack{\delta_1=\delta_2\\ \delta_3\neq\delta_4}} T^{(N=2)}_{\underline{\delta}} \\
F^{\pm}_{S} &= \sum_{\substack{\delta_1\neq\delta_2\\ \delta_3=\delta_4}} (-1)^{\delta_1} T^{(N=2)}_{\underline{\delta}} \pm i \sum_{\substack{\delta_1\neq\delta_2\\ \delta_3\neq\delta_4}} (-1)^{\delta_3} T^{(N=2)}_{\underline{\delta}} \,,
\end{aligned}
$$

where the index indicates the U(1) current (P, Q, R or S) this field is charged with. The respective holomorphic and antiholomorphic charges are $(\pm1,\pm1)$ for fields $E^{\pm}_{\cdot}$ and $(\pm1,\mp1)$ for fields $F^{\pm}_{\cdot}$. The charges w.r.t. the respective other three currents as well as J and A vanish. The following ten fields are invariant under the $\mathbb{Z}_4$ action (3.1) and are, hence, contained in $\mathbb{Z}_4(\frac{1}{\sqrt{2}}D_4, B^*)$ as well

$$
E^{\pm}_{P},\ F^{\pm}_{P}\,; \qquad E^{\pm}_{Q}\,; \qquad E^{\pm}_{R}\,; \qquad E^{\pm}_{S}\,.
$$

3.5.2 $\mathbb{Z}_4$ twistfields

In the case of $\mathbb{Z}_4$ twistfields the vertex operator coupling amounts to

$$
g^{(N=4)\,\prime}_{l}(\alpha,0) = \frac{1}{8} \qquad \forall\, l = 1,3\,.
$$

The total coupling constant $g^{(N=4)}(\alpha,0)$ still depends on the order of the twist as we will see below. The fixpoint of the twistfield is translated due to the action of the vertex operator

$$
x_f \mapsto x_{f'} = x_f + (1-\theta_4^l)^{-1}\lambda = x_f + \frac{1}{2}(1+\theta_4^l)\frac{\alpha}{\sqrt{2}}\,.
$$

Hence we get an OPE for $l=1$ of

$$
\begin{aligned}
V_{e_i+e_j}(z)\; T_f^{(N=4)\;l=1}(w) &= (V^t_{e_i+e_j}(z) + V^t_{-e_i-e_j}(z))\; T_f^{(N=4)\;l=1}(w) \\
&= g^{(N=4)\,\prime}(\alpha,0)\; e^{\frac{1}{2}\pi i(\alpha+B^*\alpha)^t\frac{1}{2}(1+\theta_4)\alpha}(z-w)^{-1} \times \\
&\quad \times \left(e^{i\pi(\alpha+B^*\alpha)^t(\sqrt{2}x_f)} + e^{-i\pi(\alpha+B^*\alpha)^t(\sqrt{2}x_f)}\right) T_{f'}^{(N=4)\;l=1}(w) \\
&= \frac{1}{4} e^{i\pi(\alpha+B^*\alpha)^t(\sqrt{2}x_f)} e^{\frac{i\pi}{2}\alpha^t L_1\alpha}\, T^{(N=4)\;l=1}_{[x_f+\frac{1}{2\sqrt{2}}(1+\theta_4)\alpha]}(w)
\end{aligned}
$$

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as well as for $l = 3$ of

$$\begin{aligned} V_{e_i+e_j}(z)\, T_f^{(N=4)\ l=3}(w) &= (V^t_{e_i+e_j}(z) + V^t_{-e_i-e_j}(z))\, T_f^{(N=4)\ l=3}(w) \\ &= g^{(N=4)\ \prime}(\alpha, 0)\ e^{\frac{1}{2}\pi i(\alpha+B^*\alpha)^t \frac{1}{2}(1+\theta_4)\alpha}(z-w)^{-1} \times \\ &\quad \times \left(e^{3i\pi(\alpha+B^*\alpha)^t(\sqrt{2}x_f)} + e^{-3i\pi(\alpha+B^*\alpha)^t(\sqrt{2}x_f)}\right) T_{f'}^{(N=4)\ l=3}(w) \\ &= \frac{1}{4} e^{3i\pi(\alpha+B^*\alpha)^t(\sqrt{2}x_f)} e^{\frac{i\pi}{2}\alpha^t L_3 \alpha} T^{(N=4)\ l=3}_{[x_f+\frac{1}{2\sqrt{2}}(1-\theta_4)\alpha]}(w) \end{aligned}$$

with

$$L_1 = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad L_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

Using this OPE one can diagonalise the twistfields according to

	q_A	q_Q	q_R	q_S
$N_1 := (T^1_{0000} + T^1_{1100}) + i(T^1_{1111} - T^1_{0011})$	$+\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$
$N_2 := (T^1_{0000} + T^1_{1100}) - i(T^1_{1111} - T^1_{0011})$	$+\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$
$N_3 := (T^1_{0000} - T^1_{1100}) + i(T^1_{1111} + T^1_{0011})$	$+\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$
$N_4 := (T^1_{0000} - T^1_{1100}) - i(T^1_{1111} + T^1_{0011})$	$+\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$
$N_5 := (T^3_{0000} + T^3_{1100}) + i(T^3_{1111} - T^3_{0011})$	$-\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$
$N_6 := (T^3_{0000} + T^3_{1100}) - i(T^3_{1111} - T^3_{0011})$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$
$N_7 := (T^3_{0000} - T^3_{1100}) + i(T^3_{1111} + T^3_{0011})$	$-\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$
$N_8 := (T^3_{0000} - T^3_{1100}) - i(T^3_{1111} + T^3_{0011})$	$-\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$

where all of these are not charged under J and P and the antiholomorphic charges are just the same as the holomorphic ones. The charge w.r.t. A cannot be derived from the above OPE, as it is a current made up of the $(1/2, 0)$ fermions of the torus theory. In appendix B we show that a very careful calculation of the R–Sector partition function — keeping the factors originating from the bosonic and fermionic characters well apart — reveals that the $\mathbb{Z}_4$ groundstate twistfields are made up of a bosonic twistfield of conformal weight $h_b = 3/16$ and a fermionic twistfield of conformal weight $h_f = 1/16$. This perfectly well corresponds to the general formulae for bosonic twistfields $h_b = \frac{1}{2}\frac{k}{N}(1 - k/N)$ and the one for fermionic twistfields $h_f = \frac{1}{2}\frac{k}{N}$ derived in [14] by CFT methods. But one also observes, inspecting the partition function, that these fields are uncharged w.r.t. the other current J made up of $(1/2, 0)$ fermions. Hence they have to be charged under A. To see this we complexify the torus fields as in 3.1 and translate our specific orbifold action (3.4) to the complexified torus. Then the orbifold action acts on both complex dimensions of the torus in just the opposite way, i.e. with multiplication with phases which are complex conjugate to each other. (This corresponds to the fact that the first two real dimensions are rotated just the opposite way as the last two by (3.4).) However, following the derivation in [14], that means that in the above formula for the twisted fermionic conformal weights h_f we have to take k for the first dimension and $-k$ for the second. But J and A, as defined in (3.2), measure the fermionic content in both complex dimensions independently (with currents of the form $\psi_+^{(i)}\psi_-^{(i)}$ in the complex dimension i). J adds the fermionic content, A subtracts it. Knowing that (by convention) l in T^l refers to the first complex dimension the claimed charges follow.

3.6 Twisted groundstate $(1/4, 1/4)$ fields in $\mathcal{K}(\mathbb{Z}_4, 0)$

This time we only have $\mathbb{Z}_2$ twistfields with generator θ_2 (3.4). The lattice vectors are given by e_i, and hence the fixpoint of the twistfield is translated due to the action of the vertex operator

$$x_f \mapsto x_{f'} = x_f + (1-\theta_2)^{-1} e_i = x_f + \frac{1}{2} e_i \,.$$

The fixpoints are elements of the finite group $x_f \in \frac{1}{2}\mathbb{Z}^4/\mathbb{Z}^4$. The coupling is given as above

$$g^{(N=2)}(\alpha, 0) = \frac{1}{4}\, e^{\frac{1}{2}\pi i (\sqrt{2} e_i)^t \frac{1}{2}(\sqrt{2} e_i)} = \frac{i}{4} \,.$$

It follows an OPE (with $\Sigma_f = T_f^{(N=2)}$)

$$\begin{aligned} U_i(z)\,\Sigma_f(w) &= (V^t_{e_i,e_i}(z) + V^t_{-e_i,-e_i}(z))\,\Sigma_f(w) \\ &= g^{(N=2)}(\alpha,0)\ (\sqrt{2}e_i, 0)(z-w)^{-1} \times \\ &\quad \times \left(e^{2\pi i (\frac{1}{\sqrt{2}} e_i)^t (\sqrt{2} x_f)} + e^{2\pi i (-\frac{1}{\sqrt{2}} e_i)^t (\sqrt{2} x_f)} \right) \Sigma_{f'}(w) \\ &= \frac{i}{2} e^{i\pi e_i^t x_f}\ \Sigma_{[x_f + \frac{1}{2} e_i]}(w) \,. \end{aligned}$$

This OPE yields the following diagonalisation of the twistfields ($N'_{i/j}$ means: N'_i resp. N'_j)

	q_P	q_Q	q_R	q_S
$E'^{\pm}_P := (\Sigma_{0000} - \Sigma_{1100} - \Sigma_{0011} + \Sigma_{1111} - \Sigma_{1010} - \Sigma_{1001} - \Sigma_{0110} - \Sigma_{0101}) \pm$ $\pm i(\Sigma_{1000} + \Sigma_{0100} + \Sigma_{0010} + \Sigma_{0001} - \Sigma_{1110} - \Sigma_{1101} - \Sigma_{1011} - \Sigma_{0111})$	± 1	0	0	0
$E'^{\pm}_Q := (\Sigma_{0000} - \Sigma_{1100} - \Sigma_{0011} + \Sigma_{1111} + \Sigma_{1010} + \Sigma_{1001} + \Sigma_{0110} + \Sigma_{0101}) \pm$ $\pm i(\Sigma_{1000} + \Sigma_{0100} - \Sigma_{0010} - \Sigma_{0001} + \Sigma_{1110} + \Sigma_{1101} - \Sigma_{1011} - \Sigma_{0111})$	0	± 1	0	0
$E'^{\pm}_R := (\Sigma_{0000} + \Sigma_{1100} + \Sigma_{0011} + \Sigma_{1111} - \Sigma_{1010} + \Sigma_{1001} + \Sigma_{0110} - \Sigma_{0101}) \pm$ $\pm i(\Sigma_{1000} - \Sigma_{0100} + \Sigma_{0010} - \Sigma_{0001} + \Sigma_{1110} - \Sigma_{1101} + \Sigma_{1011} - \Sigma_{0111})$	0	0	± 1	0
$E'^{\pm}_S := (\Sigma_{0000} + \Sigma_{1100} + \Sigma_{0011} + \Sigma_{1111} + \Sigma_{1010} - \Sigma_{1001} - \Sigma_{0110} + \Sigma_{0101}) \pm$ $\pm i(\Sigma_{1000} - \Sigma_{0100} - \Sigma_{0010} + \Sigma_{0001} - \Sigma_{1110} + \Sigma_{1101} + \Sigma_{1011} - \Sigma_{0111})$	0	0	0	± 1
$N'_{1/6} := (\Sigma_{0000} - \Sigma_{1100} + \Sigma_{0011} - \Sigma_{1111} - \Sigma_{1010} + \Sigma_{1001} - \Sigma_{0110} + \Sigma_{0101}) \pm$ $\pm i(\Sigma_{1000} + \Sigma_{0100} + \Sigma_{0010} - \Sigma_{0001} - \Sigma_{1110} + \Sigma_{1101} + \Sigma_{1011} + \Sigma_{0111})$	$\pm\frac{1}{2}$	$\pm\frac{1}{2}$	$\pm\frac{1}{2}$	$\mp\frac{1}{2}$
$N'_{3/8} := (\Sigma_{0000} + \Sigma_{1100} - \Sigma_{0011} - \Sigma_{1111} - \Sigma_{1010} - \Sigma_{1001} + \Sigma_{0110} + \Sigma_{0101}) \pm$ $\pm i(\Sigma_{1000} - \Sigma_{0100} + \Sigma_{0010} + \Sigma_{0001} + \Sigma_{1110} + \Sigma_{1101} - \Sigma_{1011} + \Sigma_{0111})$	$\pm\frac{1}{2}$	$\mp\frac{1}{2}$	$\pm\frac{1}{2}$	$\pm\frac{1}{2}$
$N'_{2/5} := (\Sigma_{0000} - \Sigma_{1100} + \Sigma_{0011} - \Sigma_{1111} + \Sigma_{1010} - \Sigma_{1001} + \Sigma_{0110} - \Sigma_{0101}) \pm$ $\pm i(\Sigma_{1000} + \Sigma_{0100} - \Sigma_{0010} + \Sigma_{0001} + \Sigma_{1110} - \Sigma_{1101} + \Sigma_{1011} + \Sigma_{0111})$	$\pm\frac{1}{2}$	$\pm\frac{1}{2}$	$\mp\frac{1}{2}$	$\pm\frac{1}{2}$
$N'_{4/7} := (\Sigma_{0000} + \Sigma_{1100} - \Sigma_{0011} - \Sigma_{1111} + \Sigma_{1010} + \Sigma_{1001} - \Sigma_{0110} - \Sigma_{0101}) \pm$ $\pm i(-\Sigma_{1000} + \Sigma_{0100} + \Sigma_{0010} + \Sigma_{0001} + \Sigma_{1110} + \Sigma_{1101} + \Sigma_{1011} - \Sigma_{0111})$	$\pm\frac{1}{2}$	$\mp\frac{1}{2}$	$\mp\frac{1}{2}$	$\mp\frac{1}{2}$

where all of these are not charged under J and A and the antiholomorphic charges are just the same as the holomorphic ones.

3.7 Fields of the Gepner model $(\hat{2})^4$

It is easier to describe the field content of the more symmetric Gepner model $(\tilde{2})^4$ with $\hat{\mathfrak{su}}(2)_1^6$ symmetry algebra first and then to derive the field content of $(\hat{2})^4$ by orbifolding. Details about the calculations in Gepner models and especially about how to compute the (2) superminimal model as a tensor product of a $c = 1$ theory and an Ising model can be found in [6]. Following [5], we define X_{ij} to be the Gepner model field with $\Phi^0_{4,2;0,0}$ as the ith and jth tensor factors and $\Phi^0_{0,0;0,0}$ elsewhere, and Y_{ij} to be the Gepner Model field with $\Phi^0_{-2,2;0,0}$ as the ith and jth tensor factors and $\Phi^0_{2,2;0,0}$ elsewhere. Furthermore let J_i be the U(1) current of the ith minimal model. Then the complete $\hat{\mathfrak{su}}(2)_1^6$ symmetry algebra of $(\tilde{2})^4$ is given by

$$
\begin{aligned}
&J'' := J_1 + J_2 + J_3 + J_4 && J''^{\pm} := \sqrt{2}\,(\Phi^0_{\mp 2,2;0,0})^{\otimes 4} \\
&A'' := J_1 + J_2 - J_3 - J_4 && A''^{+} := \sqrt{2}\,Y_{12} \quad A''^{-} := \sqrt{2}\,Y_{34} \\
&P'' := J_1 - J_2 + J_3 - J_4 && P''^{+} := \sqrt{2}\,Y_{13} \quad P''^{-} := \sqrt{2}\,Y_{24} \\
&Q'' := J_1 - J_2 - J_3 + J_4 && Q''^{+} := \sqrt{2}\,Y_{14} \quad Q''^{-} := \sqrt{2}\,Y_{23} \\
&R'' := i(X_{13} - X_{24}) && R''^{\pm} := \frac{i}{\sqrt{2}}(X_{14} + X_{23}) \pm \frac{1}{\sqrt{2}}(X_{12} + X_{34}) \\
&S'' := i(X_{13} + X_{24}) && S''^{\pm} := \frac{i}{\sqrt{2}}(X_{14} - X_{23}) \mp \frac{1}{\sqrt{2}}(X_{12} - X_{34})\,,
\end{aligned}
\tag{3.6}
$$

normalised as (3.3).

The $(1/4, 1/4)$ fields corresponding to Ramond groundstates can thus be diagonalised w.r.t. the action of the above currents

$$
\begin{aligned}
E''^{\pm}_J &= (\Phi^0_{\mp 1,\mp 1;\mp 1,\mp 1})^{\otimes 4} \\
F''^{\pm}_J &= (\Phi^0_{\mp 1,\mp 1;\pm 1,\pm 1})^{\otimes 4} \\
E''^{\pm}_A &= (\Phi^0_{\mp 1,\mp 1;\mp 1,\mp 1})^{\otimes 2} \otimes (\Phi^0_{\pm 1,\pm 1;\pm 1,\pm 1})^{\otimes 2} \\
F''^{\pm}_A &= (\Phi^0_{\mp 1,\mp 1;\pm 1,\pm 1})^{\otimes 2} \otimes (\Phi^0_{\pm 1,\pm 1;\mp 1,\mp 1})^{\otimes 2} \\
E''^{\pm}_P &= \Phi^0_{\mp 1,\mp 1;\mp 1,\mp 1} \otimes \Phi^0_{\pm 1,\pm 1;\pm 1,\pm 1} \otimes \Phi^0_{\mp 1,\mp 1;\mp 1,\mp 1} \otimes \Phi^0_{\pm 1,\pm 1;\pm 1,\pm 1} \\
F''^{\pm}_P &= \Phi^0_{\mp 1,\mp 1;\pm 1,\pm 1} \otimes \Phi^0_{\pm 1,\pm 1;\mp 1,\mp 1} \otimes \Phi^0_{\mp 1,\mp 1;\pm 1,\pm 1} \otimes \Phi^0_{\pm 1,\pm 1;\mp 1,\mp 1} \\
E''^{\pm}_Q &= \Phi^0_{\mp 1,\mp 1;\mp 1,\mp 1} \otimes \Phi^0_{\pm 1,\pm 1;\pm 1,\pm 1} \otimes \Phi^0_{\pm 1,\pm 1;\pm 1,\pm 1} \otimes \Phi^0_{\mp 1,\mp 1;\mp 1,\mp 1} \\
F''^{\pm}_Q &= \Phi^0_{\mp 1,\mp 1;\pm 1,\pm 1} \otimes \Phi^0_{\pm 1,\pm 1;\mp 1,\mp 1} \otimes \Phi^0_{\pm 1,\pm 1;\mp 1,\mp 1} \otimes \Phi^0_{\mp 1,\mp 1;\pm 1,\pm 1} \\
E''^{\pm}_R &= (\Phi^1_{2,1;2,1})^{\otimes 4} + (\Phi^1_{2,1;-2,-1})^{\otimes 4} \pm \\
&\quad \pm \left[(\Phi^1_{2,1;2,1} \otimes \Phi^1_{2,1;-2,-1})^{\otimes 2} - (\Phi^1_{2,1;-2,-1} \otimes \Phi^1_{2,1;2,1})^{\otimes 2}\right] \\
F''^{\pm}_R &= (\Phi^1_{2,1;2,1})^{\otimes 2} \otimes (\Phi^1_{2,1;-2,-1})^{\otimes 2} + (\Phi^1_{2,1;-2,-1})^{\otimes 2} \otimes (\Phi^1_{2,1;2,1})^{\otimes 2} \pm \\
&\quad \pm i\Big[\Phi^1_{2,1;-2,-1} \otimes \Phi^1_{2,1;2,1} \otimes \Phi^1_{2,1;2,1} \otimes \Phi^1_{2,1;-2,-1} + \\
&\qquad + \Phi^1_{2,1;2,1} \otimes \Phi^1_{2,1;-2,-1} \otimes \Phi^1_{2,1;-2,-1} \otimes \Phi^1_{2,1;2,1}\Big]
\end{aligned}
$$

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$$
\begin{aligned}
E''^{\pm}_{S} &= (\Phi^1_{2,1;2,1})^{\otimes 4} - (\Phi^1_{2,1;-2,-1})^{\otimes 4} \mp \\
&\quad \mp \left[(\Phi^1_{2,1;2,1} \otimes \Phi^1_{2,1;-2,-1})^{\otimes 2} + (\Phi^1_{2,1;-2,-1} \otimes \Phi^1_{2,1;2,1})^{\otimes 2}\right] \\
F''^{\pm}_{S} &= (\Phi^1_{2,1;2,1})^{\otimes 2} \otimes (\Phi^1_{2,1;-2,-1})^{\otimes 2} - (\Phi^1_{2,1;-2,-1})^{\otimes 2} \otimes (\Phi^1_{2,1;2,1})^{\otimes 2} \pm \\
&\quad \pm i \Big[\Phi^1_{2,1;-2,-1} \otimes \Phi^1_{2,1;2,1} \otimes \Phi^1_{2,1;2,1} \otimes \Phi^1_{2,1;-2,-1} - \\
&\qquad - \Phi^1_{2,1;2,1} \otimes \Phi^1_{2,1;-2,-1} \otimes \Phi^1_{2,1;-2,-1} \otimes \Phi^1_{2,1;2,1}\Big] .
\end{aligned}
$$

As in section 3.5 the index indicates the U(1) current this field is charged with, holomorphic resp. antiholomorphic charges $(\pm 1, \pm 1)$ for $E^{\pm}_{\cdot}$ and $(\pm 1, \mp 1)$ for $F^{\pm}_{\cdot}$.

Now, as $(\hat{2})^4$ is gained from $(\tilde{2})^4$ by an $\mathbb{Z}_2$ orbifold generated by

$$
[2', 2', 0, 0] : \quad \bigotimes_{i=1}^{4} \Phi^{l_i}_{m_i, s_i; \bar{m}_i, \bar{s}_i} \;\mapsto\; e^{\frac{2\pi i}{8}(\bar{m}_1 - m_1 - \bar{m}_3 + m_3)} \bigotimes_{i=1}^{4} \Phi^{l_i}_{m_i, s_i; \bar{m}_i, \bar{s}_i} ,
$$

the surviving $\hat{\mathfrak{su}}(2)^2_1 \otimes \hat{\mathfrak{u}}(1)^4$ symmetry algebra of $(\hat{2})^4$ is given by the currents

$$
J,\ J^{\pm}; \quad A; \quad P,\ P^{\pm}; \quad Q; \quad R; \quad S .
$$

Of the above Ramond groundstate fields the following are invariant under the orbifold group action

$$
E''^{\pm}_{J},\ F''^{\pm}_{J}; \quad E''^{\pm}_{A}; \quad E''^{\pm}_{P},\ F''^{\pm}_{P}; \quad E''^{\pm}_{Q}; \quad E''^{\pm}_{R}; \quad E''^{\pm}_{S} .
$$

The list of $(1/4, 1/4)$ fields in $(\hat{2})^4$ has to be completed by the following twistfields (w.r.t. the above orbifold construction)

$$
\begin{aligned}
T_1 &= \Phi^1_{2,1;2,1} \otimes \Phi^0_{-1,-1;-1,-1} \otimes \Phi^1_{2,1;2,1} \otimes \Phi^0_{1,1;1,1} \\
T_2 &= \Phi^1_{2,1;2,1} \otimes \Phi^0_{1,1;1,1} \otimes \Phi^1_{2,1;2,1} \otimes \Phi^0_{-1,-1;-1,-1} \\
T_3 &= \Phi^0_{-1,-1;-1,-1} \otimes \Phi^1_{2,1;2,1} \otimes \Phi^0_{1,1;1,1} \otimes \Phi^1_{2,1;2,1} \\
T_4 &= \Phi^0_{1,1;1,1} \otimes \Phi^1_{2,1;2,1} \otimes \Phi^0_{-1,-1;-1,-1} \otimes \Phi^1_{2,1;2,1} \\
T_5 &= \Phi^1_{2,1;-2,-1} \otimes \Phi^0_{-1,-1;-1,-1} \otimes \Phi^1_{2,1;-2,-1} \otimes \Phi^0_{1,1;1,1} \\
T_6 &= \Phi^1_{2,1;-2,-1} \otimes \Phi^0_{1,1;1,1} \otimes \Phi^1_{2,1;-2,-1} \otimes \Phi^0_{-1,-1;-1,-1} \\
T_7 &= \Phi^0_{-1,-1;-1,-1} \otimes \Phi^1_{2,1;-2,-1} \otimes \Phi^0_{1,1;1,1} \otimes \Phi^1_{2,1;-2,-1} \\
T_8 &= \Phi^0_{1,1;1,1} \otimes \Phi^1_{2,1;-2,-1} \otimes \Phi^0_{-1,-1;-1,-1} \otimes \Phi^1_{2,1;-2,-1} .
\end{aligned}
$$

These can be diagonalised w.r.t. to the action of the invariant U(1) currents

	q_J	q_A	q_P	q_Q	q_R	q_S
$N''_{1/2} = T_3 \pm T_7$	0	$+\frac{1}{2}$	0	$+\frac{1}{2}$	$\pm\frac{1}{2}$	$\pm\frac{1}{2}$
$N''_{3/4} = T_1 \mp T_5$	0	$+\frac{1}{2}$	0	$-\frac{1}{2}$	$\pm\frac{1}{2}$	$\pm\frac{1}{2}$
$N''_{5/6} = T_4 \pm T_8$	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$\pm\frac{1}{2}$	$\pm\frac{1}{2}$
$N''_{7/8} = T_2 \mp T_6$	0	$-\frac{1}{2}$	0	$+\frac{1}{2}$	$\pm\frac{1}{2}$	$\pm\frac{1}{2}$

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3.8 Explicit identification of the three theories

Now we can perform the identification of the three CFTs $(\hat{2})^4$, $\mathcal{K}(\mathbb{Z}^4,0)$ and $\mathbb{Z}_4(\frac{1}{\sqrt{2}}D_4,B^*)$, already proven in [5], explicitly for the symmetry algebra and the lattice of $(1/4,1/4)$ fields. The identification of the symmetry algebra is not totally fixed (this reflects the high amount of symmetry of these theories); one possible way of identifying the currents is

$$
\begin{aligned}
J &\simeq J'' \simeq J' \qquad J^\pm \simeq J''^\pm \simeq J'^\pm \\
A &\simeq A'' \simeq P' \\
P &\simeq P'' \simeq A' \qquad P^\pm \simeq P''^\pm \simeq A'^\pm \\
Q &\simeq Q'' \simeq Q' \\
R &\simeq R'' \simeq R' \\
S &\simeq S'' \simeq S' \,.
\end{aligned}
$$

Comparing the charges w.r.t. the above currents this leads to the following identification of $(1/4,1/4)$ fields

$$
\begin{aligned}
E_J^\pm &\simeq E_J''^\pm \simeq E_J'^\pm \qquad F_J^\pm \simeq F_J''^\pm \simeq F_J'^\pm \\
E_A^\pm &\simeq E_A''^\pm \simeq E_P'^\pm \\
E_P^\pm &\simeq E_P''^\pm \simeq E_A'^\pm \qquad F_P^\pm \simeq F_P''^\pm \simeq F_A'^\pm \\
E_Q^\pm &\simeq E_Q''^\pm \simeq E_Q'^\pm \\
E_R^\pm &\simeq E_R''^\pm \simeq E_R'^\pm \\
E_S^\pm &\simeq E_S''^\pm \simeq E_S'^\pm \\
N_i &= N_i' = N_i'' \qquad \forall\, i \in \{1,\dots,8\}\,.
\end{aligned} \tag{3.7}
$$

This identification leads to the important observation that the $\mathbb{Z}_2$ and $\mathbb{Z}_4$ orbifold subvarieties in K3 moduli space which intersect in one point as described earlier are orthogonal to each other. In order to see this it is important to recall some facts about conformal deformation theory [8, 14, 6]. The possible exact marginal deformation fields in these $c=6$ theories are not charged under the U(1) current J of the SUSY algebra and can be generated as the $(1,1)$ superpartners of $(1/2,1/2)$ NS fields which again can be generated via spectral flow from $(1/4,1/4)$ Ramond fields. Now we have two types of deformation fields within our orbifold theories. Deformation fields which originate from the original torus theory generate all the deformations along the orbifold subvarieties and are well understood. The corresponding $(1/4,1/4)$ Ramond fields in $\mathcal{K}(\mathbb{Z}^4,0)$ are $E_A'^\pm$ and $F_A'^\pm$, the ones in $\mathbb{Z}_4(\frac{1}{\sqrt{2}}D_4,B^*)$ are $E_A^\pm$. On the other hand, we do not understand very much about deformations with twistfields. The $(1/4,1/4)$ Ramond fields corresponding to these deformations are given by $E_P'^\pm$, $E_Q'^\pm$, $E_R'^\pm$, $E_S'^\pm$, N_i' in $\mathcal{K}(\mathbb{Z}^4,0)$, and $E_P^\pm$, $F_P^\pm$, $E_Q^\pm$, $E_R^\pm$, $E_S^\pm$, N_i in $\mathbb{Z}_4(\frac{1}{\sqrt{2}}D_4,B^*)$. Now the orbifold group selection rules imply that the deformation fields originating from the torus theory and the twisted deformation fields are orthogonal w.r.t. the Zamolodchikov metric as any two point function of the two types has to vanish. Now the identification in (3.7) implies that torus deformations of one theory are identified with twistfield deformations of the respective other. This, the fact that deformations along the orbifold subvarieties are generated by torus deformations and the orthogonality of the two types of deformation fields prove the above stated orthogonality of the $\mathbb{Z}_2$ and $\mathbb{Z}_4$ orbifold subvarieties.

4. Conclusion

In this paper we have given an explicit identification of the coordinates of the $\mathbb{Z}_2$ and $\mathbb{Z}_4$ orbifolds at the intersection of the $\mathbb{Z}_2$ and $\mathbb{Z}_4$ orbifold subvarieties in the K3 moduli space. This is an important step towards the exploration of the unknown parts of the K3 moduli space via conformal deformation theory. It enables us to relate the coordinates of the starting and end points of geodesics running between the $\mathbb{Z}_2$ and $\mathbb{Z}_4$ orbifold subvarieties which we suppose to be the most promising lines for a successful finite deformation. Open questions in this direction are the explicit relation between the geometric deformations and the deformation fields of the respective conformal field theories as well as the calculation of the deformed conformal weights and structure constants of the CFTs themselves. Some results in this direction are presented in [9]; both questions are challenging, though, and we are currently pursuing them further.

Furthermore, elaborating the explicit identification of the respective CFTs at this point, especially on the level of the symmetry algebra and the groundstate lattice, we have shown the important fact that the two subvarieties are orthogonal to each other. As we can now express deformation twistfields of one orbifold theory by torus deformation fields in another at this point of intersection, we can use this to probe the deformation by twistfields. But twistfield deformations are exactly these deformation fields we need to explore the unknown parts of the K3 moduli space, as described above. Hence, we may hope to find some clues at this intersection point for the above mentioned unsolved problem how to identify the geometric deformations and the conformal deformation fields. Additionally, this identification might also help to clarify the peculiar symmetry in the dependence of the deformation of a vertex operator in an orbifold model on the conformal dimension about the point $h = 1/8$, observed in [9].

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A. Explicit identification of lattice vectors

In order to prove the theorem in section 2.4 we have explicitely checked the identification for all 42 different generators of $\Gamma(\mathcal{K}(\mathbb{Z}^4, 0))$ described in the equations (2.1) and (2.2) (with lattice $\Lambda = \mathbb{Z}^4$). In this appendix we collect a variety of typical examples of this calculation. Each time we first express a generator of $\Gamma(\mathcal{K}(\mathbb{Z}^4, 0))$ in terms of the orthogonal basis of A_i, then we use the identification given in section 2.4 to rewrite this expression

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in terms of the B_j, and finally we express this in terms of a sum of lattice vectors of $\Gamma(\mathbb{Z}_4(\frac{1}{\sqrt{2}}D_4, B^*))$:

$$
\begin{aligned}
\hat{E}_{0000} &= \frac{1}{4}(A_5 + A_6 + A_7 + A_8 + A_{13} + A_{14} + A_{15} + A_{16} + A_4 - A_{24}) \\
&\widehat{=} \frac{1}{4}(B_2 - B_{13} + B_{14} + B_{16} - B_9 - B_{10} + B_{11} + B_{12} - B_{15} - B_{24}) \\
&= -\hat{\upsilon} - \hat{E}^{(+)}_{1100} - \hat{E}^{(0)}_{1100} - \hat{E}^{(-)}_{1100} \\
\hat{E}_{1000} &= \frac{1}{4}(A_9 + A_{10} + A_{11} + A_{12} + A_{17} + A_{18} + A_{19} + A_{20} + A_4 - A_{24}) \\
&\widehat{=} \frac{1}{4}(-B_1 + B_3 + B_8 + B_7 - B_{17} - B_{18} + B_{19} - B_{20} - B_{15} - B_{24}) \\
&= \left(-e_1 \wedge e_2 + \frac{1}{2}(\hat{E}^{(+)}_{0000} + \hat{E}^{(-)}_{0000} + \hat{E}^{(+)}_{0011} + \hat{E}^{(-)}_{0011})\right) + \\
&\quad + \Bigg(\frac{1}{2}e_3 \wedge e_4 - \frac{1}{2}\hat{E}_{0100} - \frac{1}{4}(-\hat{E}^{(+)}_{0000} - 2\hat{E}^{(0)}_{0000} - 3\hat{E}^{(-)}_{0000} - \hat{E}^{(+)}_{1100} - \\
&\qquad - 2\hat{E}^{(0)}_{1100} - 3\hat{E}^{(-)}_{1100})\Bigg) + \\
&\quad + \left(-\frac{1}{2}(e_1 \wedge e_3 + e_4 \wedge e_2) - \frac{1}{2}(\hat{E}^{(+)}_{0000} + \hat{E}^{(-)}_{0000}) + \frac{1}{2}(\hat{E}_{0100} + \hat{E}_{0001} + \hat{E}_{0101})\right) + \\
&\quad + \left(\frac{1}{2}(e_1 \wedge e_4 + e_2 \wedge e_3) + \frac{1}{2}(\hat{E}^{(+)}_{0011} + \hat{E}^{(-)}_{0011}) + \frac{1}{2}(-\hat{E}_{0111} - \hat{E}_{0001} - \hat{E}_{0101})\right) - \\
&\quad - \hat{\upsilon}^\circ - \hat{E}^{(+)}_{0000} - \hat{E}^{(0)}_{0000} - \hat{E}^{(-)}_{0000} - \hat{E}^{(+)}_{1100} - \hat{E}^{(0)}_{1100} - \hat{E}^{(-)}_{1100} - \hat{E}^{(+)}_{0011} - \hat{E}^{(-)}_{0011} \\
\frac{1}{\sqrt{2}}&e_1 \wedge e_2 + \frac{1}{2}(\hat{E}_{0000} - \hat{E}_{0010} - \hat{E}_{0001} + \hat{E}_{0011}) = \\
&= \frac{1}{4}(A_1 + A_{21} + A_7 + A_8 + A_{13} + A_{14} - A_{19} + A_{20} - A_9 + A_{10}) \\
&\widehat{=} \frac{1}{4}(B_6 - B_{21} + B_{14} + B_{16} - B_9 - B_{10} - B_{19} - B_{20} + B_1 + B_3) \\
&= \Bigg(-\frac{1}{2}e_3 \wedge e_4 + \frac{1}{2}\hat{E}_{0100} + \frac{1}{4}(-\hat{E}^{(+)}_{0000} - 2\hat{E}^{(0)}_{0000} - 3\hat{E}^{(-)}_{0000} - \hat{E}^{(+)}_{1100} - \\
&\qquad - 2\hat{E}^{(0)}_{1100} - 3\hat{E}^{(-)}_{1100})\Bigg) + \hat{E}^{(-)}_{0000} \\
\hat{\upsilon} &= \frac{1}{2}(-A_4 + A_{24}) \cong \frac{1}{2}(B_{15} + B_{24}) \\
&\widehat{=} \left(e_1 \wedge e_2 + \frac{1}{2}(\hat{E}^{(+)}_{0000} + \hat{E}^{(-)}_{0000} + \hat{E}^{(+)}_{0011} + \hat{E}^{(-)}_{0011})\right) + \\
&\quad + \left(\frac{1}{2}(e_1 \wedge e_3 + e_4 \wedge e_2) - \frac{1}{2}(\hat{E}^{(+)}_{0000} + \hat{E}^{(-)}_{0000}) + \frac{1}{2}(\hat{E}_{0100} + \hat{E}_{0001} + \hat{E}_{0101})\right) + \\
&\quad + \left(-\frac{1}{2}(e_1 \wedge e_4 + e_2 \wedge e_3) + \frac{1}{2}(\hat{E}^{(+)}_{1111} + \hat{E}^{(-)}_{1111}) + \frac{1}{2}(\hat{E}_{0111} + \hat{E}_{1110} + \hat{E}_{0110})\right) + \\
&\quad + 2\hat{\upsilon}^\circ + 2\hat{\upsilon} + \sum_{i \in \{0000,1100\}} (\hat{E}^{(+)}_i + \hat{E}^{(0)}_i + \hat{E}^{(0)}_i) + \hat{E}^{(0)}_{0011} + \hat{E}^{(0)}_{1111}
\end{aligned}
$$

$$
\begin{aligned}
\hat{v}^\circ &= \frac{1}{4}(-A_4 + A_5 - A_6 - A_7 - A_8 - A_{17} - A_{18} - A_{19} + A_{20} + 3A_{24}) \\
&\widehat{=} \frac{1}{4}(B_{15} + B_2 + B_{13} - B_{14} - B_{16} + B_{17} + B_{18} - B_{19} - B_{20} + 3B_{24}) \\
&= 4\hat{v}^\circ + 4\hat{v} + 2e_1 \wedge e_2 + (e_1 \wedge e_3 + e_4 \wedge e_2) - (e_1 \wedge e_4 + e_2 \wedge e_3) + \sum_{i \in I^{(2)}} \hat{E}_i + \\
&\quad + 2 \sum_{i \in I^{(4)}} \left(\hat{E}_i^{(+)} + \hat{E}_i^{(0)} + \hat{E}_i^{(-)} \right) - \frac{1}{2} \sum_{i \in I^{(4)}} \left(\hat{E}_i^{(+)} + \hat{E}_i^{(-)} \right) + \hat{E}_{1100}^{(+)} .
\end{aligned}
$$

B. R-sector of the $\mathbb{Z}_4$ orbifold partition function

Using projection and modular transformations one can calculate the partition function of a $\mathbb{Z}_4$ orbifold from its original torus theory, always keeping the characters of the bosonic and the fermionic part separate factors. We are only interested in the R-sector of this partition function

$$
\begin{aligned}
Z_{\mathbb{Z}_4}^{\Gamma,R} = \frac{1}{4} \Bigg(& \left(\frac{1}{|\eta(\sigma)|^8} \sum_{p \in \Gamma} q^{p_l^2/2} \bar{q}^{p_r^2/2} \right) * \left| \frac{\theta_2(\sigma, z)}{\eta(\sigma)} \right|^4 + 8 \left| \frac{\eta(2\sigma)}{\theta_2(2\sigma)} \right|^2 * \left| \frac{\theta_2(2\sigma, 2z)}{\eta(2\sigma)} \right|^2 + \\
& + 16 \left| \frac{\eta(\sigma)}{\theta_2(\sigma)} \right|^4 * \left| \frac{\theta_1(\sigma, z)}{\eta(\sigma)} \right|^4 + 16 \left| \frac{\eta(\sigma)}{\theta_4(\sigma)} \right|^4 * \left| \frac{\theta_3(\sigma, z)}{\eta(\sigma)} \right|^4 + \\
& + 16 \left| \frac{\eta(\sigma)}{\theta_3(\sigma)} \right|^4 * \left| \frac{\theta_4(\sigma, z)}{\eta(\sigma)} \right|^4 + 8 \left| \frac{\eta(\frac{\sigma}{2})}{\theta_4(\frac{\sigma}{2})} \right|^4 * \left| \frac{\theta_3(\frac{\sigma}{2}, z)}{\eta(\frac{\sigma}{2})} \right|^4 + \\
& + 8 \left| \frac{(\eta(\sigma)\theta_3(\sigma))^{1/2}}{\theta_3(2\sigma) - i\theta_2(2\sigma)} \right|^2 * \left| \frac{(\theta_3(2\sigma, 2z) + i\theta_2(2\sigma, 2z))}{(\eta(\sigma)\theta_3(\sigma))^{1/2}} \right|^2 + \\
& + 8 \left| \frac{(\eta(\sigma)\theta_4(\sigma))^{1/2}}{\theta_3(2\sigma) + \theta_2(2\sigma)} \right|^2 * \left| \frac{(\theta_3(2\sigma, 2z) - \theta_2(2\sigma, 2z))}{(\eta(\sigma)\theta_4(\sigma))^{1/2}} \right|^2 + \\
& + 8 \left| \frac{(\eta(\sigma)\theta_3(\sigma))^{1/2}}{\theta_3(2\sigma) + i\theta_2(2\sigma)} \right|^2 * \left| \frac{(\theta_3(2\sigma, 2z) - i\theta_2(2\sigma, 2z))}{(\eta(\sigma)\theta_3(\sigma))^{1/2}} \right|^2 + \\
& + 8 \left| \frac{(\eta(\sigma)\theta_2(\sigma))^{1/2}}{\theta_3(\frac{\sigma}{2}) + \theta_4(\frac{\sigma}{2})} \right|^2 * \left| \frac{(\theta_3(\frac{\sigma}{2}, 2z) + \theta_4(\frac{\sigma}{2}, 2z))}{(\eta(\sigma)\theta_2(\sigma))^{1/2}} \right|^2 \Bigg) ;
\end{aligned}
$$

in each term the first factor gives the bosonic part, the second the fermionic. The first three terms constitute the untwisted sector, the other seven terms the different twisted sectors. Expanding the terms six to nine yields a leading contribution of

$$
2 \left((q\bar{q})^{-1/6} (q\bar{q})^{3/16} \right) * \left((q\bar{q})^{-1/12} (q\bar{q})^{1/16} \right) * 1
$$

for each. The first big bracket gives the bosonic contribution, separating the overall modular factor first, the second bracket gives the fermionic contribution. Hence we find eight fields of overall conformal weight $(h, \bar{h}) = (1/4, 1/4)$ where a $h_b = 3/16$ part originates from bosonic degrees of freedom, a $h_f = 1/16$ part from fermionic. This perfectly well coincides with the conformal weights of twistfields generating cuts for either bosonic or fermionic fields found in [14].

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